Solving Lock Box Location Problems

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Introduction

"Cash management is being given a priority throughout U.S. business today that it has never enjoyed before, and company after company is being swept along," according to a recent *Business Week* article [4]. Need for more efficient cash management has concentrated attention on more sophisticated computer-based methods for managing corporate cash flows. During the last few years, more and more corporations have begun using computer-based management science approaches to cash management.

One of the most important aspects of cash management is development of an efficient receivables collection system that makes it possible to collect payments quickly from a number of wide-spread customers. A company that must collect payments from such customers generally maintains "lock box" accounts with banks in several strategically-located cities. The company wants to select a *set* of lock box banks to minimize both the opportunity costs of uncollected funds and lock box service charges. The increased profits resulting from such a use of lock box collection systems have been very significant to many companies [10, 15].

Researchers have dealt with the lock box problem. Levy [13] developed a heuristic procedure for the selection of a low cost combination of lock boxes from a given set of possible locations. Kramer [11] evaluated various lock box plans by simulating their mail- and clearing-time for a representative sample of checks. Stancill [24] developed a procedure for determining the costs and benefits for a given lock box. McAdams [14] provided a critique of Stancill's work, arguing for a more rigorous definition of the opportunity benefits accruing to the funds released through the use of a lock box. Kraus, Janssen, and McAdams [12] later designed an integer programming formulation of the lock box problem, but did not show any computational results. Shanker and Zoltners [23, 24] extended the formulation proposed by Kraus, Janssen, and McAdams, again with no actual computational results. More recently, Maier and Vander Weide [15, 16] have specified a different approach to this problem, also failing to present computational results. Cornuejols, Fisher, and Nemhauser [6] have produced an excellent theoretical treatise and survey of the state of the art in lock box location modeling. There are very brief discussions of computational results in working papers by Ciochetto *et al.* [5] and by Bulfin and Unger [2].

The basic objective of this article is to describe the development and application of an efficient model for solving large-scale lock box location problems. The discussion deals with the practical aspects of implementing this model, using sophisticated computer programs that provide the user with modeling flexibility. The model presented is an improvement over existing models in terms of its ability to solve large problems in a reasonable amount of computer time.

Formulation of the Lock Box Model

The lock box location problem may be defined as how to select a set of lock box banks and assign customers to those banks so as to minimize the opportunity cost of uncollected funds and lock box service charges. It is important to note that the opportunity cost of uncollected funds ($-float) is commensurate with lock box service charges whether the charges are on a cash basis or on a compensating balance basis. For example, suppose a firm could use a proposed lock box configuration to decrease $-float by $100,000 per day over its *existing* lock box configuration. At a 6% marginal rate of return for investment of corporate funds, the firm then could realize an additional return of $6,000 per year. If the lock box charges (on a cash basis) are $6,000 more than the charges for the existing configuration, however, the firm would obviously reject the new proposal.

In order to formulate the model in a detailed fashion, we define two sets of indices. Let $J = \{1,2, \ldots, n\}$ be the index set of prospective lock box banks. While it may be easier to think of each index as representing a particular city, it is quite straightforward to let each index represent a particular bank. Thus it is possible to have banks in the same city represented in the model.

Let $I = \{1, \ldots, m\}$ be the index set of customer zones in the model. In the extreme case, each corporate customer could be represented by a customer zone. Generally, though, m would be quite large. In order to make the model more manageable, customers are aggregated into customer zones. The criterion of aggregation most often used is the zip codes from which customer checks were mailed. For example, if the first two zip code digits represent a mailing area, then one hundred customer zones are formed, starting with 00 and ending with 99.

In certain instances it may be inappropriate to define zones based solely on zip codes. Consider a hypothetical situation in which two checks are mailed from two different customers in Seattle to a lock box in Portland. One of these checks is drawn on a bank in Seattle, the other on a bank located in Boston. Presumably, the total collection time will be much greater for the check drawn on the Boston bank than for the check drawn on the Seattle bank. In such a situation, it may be advantageous to have the Seattle check drawn on the bank in Seattle sent to the lock box in Portland, and to have the Seattle check drawn on the bank in Boston sent instead to a lock box in Chicago. This practice makes it possible to reduce the sum of the total collection times (and hence float) for the two checks.

Federal Reserve districts can be used to decide whether or not a customer should be placed into a zip code customer zone. Assign each two-digit zip code to the Federal Reserve district where the zip code area is located. If a customer's check mailed from a certain zip code is drawn on a bank in the same Federal Reserve district, then the customer belongs in the normal customer zip code zone. If the check is drawn on a bank not in that Federal Reserve district, it is possible to create a separate customer zone for that customer.

A customer who has two or more checking accounts in different cities, and who writes checks on these accounts in a random fashion, cannot be easily (or correctly) assigned to a particular customer zone. (In fact, such a customer may use different checking accounts to maximize the clearing time for checks. Assignment to a particular lock box would probably cause him to use the checking account that would result in the longest clearing time for that particular lock box.)

Given index sets I and J for customer zones and lock boxes respectively, we can define the following terms. Let d_j be the annual fixed cost associated with maintaining a lock box account at bank j. Let s_j be the per check processing charge at lock box bank j. Let h_i be the expected number of checks received from customer zone i. The value for h_i can be estimated from the number of customers in customer zone i, or it can be obtained (using a factor obtained from annual figures) from the sample of customer checks being used in the lock box study. Let K be the maximum number of lock boxes that can be maintained. If there is no artificial limit on the number of lock boxes, then K may be set to n. Next, let c_{ij} be the opportunity cost of $-float if customer zone i checks are sent to lock box j. (Detailed prescriptions for calculating c_{ij} are presented in the Data Calculations section.)

Finally, we define the following variables: Let y_j

equal 1 if lock box j is opened, and 0 if it is not. Let x_{ij} equal 1 if checks from customer zone i are to be sent to lock box j, and 0 if not.

We can then formulate the lock box model as an integer program as follows:

Minimize

$$Z = \sum_{i\epsilon I} \sum_{j\epsilon J} c_{ij}x_{ij} + \sum_{j\epsilon J} d_j y_j + \sum_{i\epsilon I} \sum_{j\epsilon J} h_i s_j x_{ij} \quad (1)$$

subject to:

$$\sum_{j\epsilon J} x_{ij} = 1, i\epsilon I \quad (2)$$

$$\sum_{j\epsilon J} y_j \leq K \quad (3)$$

$$y_j \epsilon \{0, 1\}, j\epsilon J \quad (4)$$

$$x_{ij} \epsilon \{0, 1\}, i\epsilon I, j\epsilon J \quad (5)$$

$$\sum_{i\epsilon I} x_{ij} \leq m y_j, j\epsilon J. \quad (6)$$

The objective function (1) that is being minimized is the sum of the opportunity cost of dollar-float of deposits made at all lock boxes, plus the fixed and variable costs associated with operating these lock boxes. Constraint (2) states that each customer zone i must be assigned to exactly one lock box j. Constraint (3) simply requires K or fewer lock boxes to be open, and constraint (6) requires that a lock box be open in order for a customer zone location to be assigned to that lock box. It should also be noted that the x_{ij} variables may be treated as continuous variables, since there always exists an optimal solution which has all x_{ij} either 0 or 1 whether or not the $x_{ij} \epsilon \{0, 1\}$ constraints are imposed.

Data Calculations

Opportunity Cost of $-float

Calculation of the opportunity cost of $-float (the c_{ij} term) may be done as follows. First, we define three components that make up total collection time or total float:

1. m_{ij}: Mail float — the period of time between the mailing of a check from customer zone i and its receipt by lock box bank j. (These data may be obtained from Phoenix-Hecht, Inc., a private firm that collects such data.)
2. p_j: Processing float — the period of time between receipt of a check by lock box bank j and its deposit.
3. r_{jl}: Clearing float — the period of time between deposit of a remittance in lock box bank j and when it clears, where l denotes the bank upon which the check is drawn. Further, r_{jl} may depend on the size of the check, a_k. Some banks expedite clearing of checks over a particular dollar amount.

To calculate the clearing float, we require the hourly mail delivery schedule, lock box (post office) pickup schedule, percentage of weekly mail received each day, and the check availability schedule for lock box j. Finally, let α be the current annual marginal interest rate for investment of corporate funds.

To illustrate, suppose two checks for $5,000 each are sent from Atlanta to St. Louis each day of the year. One of the checks is drawn on an Atlanta bank, the other on a bank in Plains. For simplicity, suppose that the same amount of mail received by the St. Louis lock box is received each hour of the day and that mail pickups at the lock box are made at 6 a.m., 2 p.m., and 4 p.m. The average mail time from Atlanta to St. Louis is two days, lock box processing time is two hours, and the lock box operation collects mail Monday through Friday (25% on Monday, 15% on Tuesday, 19% on Wednesday, 20% on Thursday, and 21% on Friday). An assumed check availability schedule for the St. Louis lock box is presented in Exhibit 1 (for Atlanta checks) and Exhibit 2 (for Plains checks). At the bottom of Exhibits 1 and 2, we present the computation of the expected total float for the assumed availability schedules. Yearly opportunity cost of dollar-float for Atlanta to St. Louis (for α = .06) is .06 (5,000) (3.900) + .06 (5,000) (5.185) = $2,725.50.

For a more general mail delivery schedule, a cumulative probability distribution, M(t) would be used, where M(t) = Pr (a piece of mail is delivered before t hundred hours). Thus, M(0) = 0, and M(24) = 1. In our example, we have M(0) = 0, M(1) = 1/24, M(2) = 2/24, . . . , M(24) = 1. Note also that processing float may or may not be included in the calculation, depending on whether or not a bank's availability schedule states that a check must be received by the cutoff time or if it must be received, say, V hours before the cutoff time. In addition, if a probability distribution for mail float is available (rather than just average mail float), as well as a distribution for when during the week letters are mailed, a more detailed and realistic opportunity cost of dollar-float figure could be calculated.

The opportunity cost of $-float for one check of amount a_k drawn on bank l and mailed from customer zone i to lock box j is

$$opc_k = \frac{\alpha}{365}(m_{ij} + p_j + r_{jl})a_k. \quad (7)$$

Now suppose that in one year T checks of size a_k, k=1, . . . , T were sent from customer zone i to lock box j. The opportunity cost of $-float for customer zone i to lock box j is

$$c_{ij} = \sum_{k=1}^{T} opc_k. \qquad (8)$$

To express c_{ij} in annual terms (assuming no seasonal effects), multiply a month's sample by 12.

Lock Box Charges

Charges for lock box services vary from bank to bank. Most charges are based on two components: a fixed charge that is independent of lock box activity and a variable charge that depends on the number of checks processed. Fixed charges are assessed on a periodic basis (daily, weekly, monthly, etc.) regardless of the number of checks processed. Examples of fixed charges include monthly account maintenance, daily deposit fees, depository transfer checks, and/or daily wire transfer charges.

One important point should be made concerning daily wire transfer charges. A single wire transfer is assessed an outgoing charge by the originating bank as well as an incoming charge by the receiving (concentration) bank. To calculate an annual figure for wire transfers for a non-concentration bank, both the originating wire transfer charge and the receiving charge by the concentration bank should be included in the non-concentration bank's annual figure. The reasoning for this is quite simple. If all customer checks were mailed directly to the concentration bank's lock box, no wire transfer charges would be incurred. Wire transfer charges are incurred only if a non-concentration bank's lock box is selected, and

Exhibit 1. Atlanta Availability Schedule

Available	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.	Mon.	
Received Monday	0	1	2	3	4	5	6	7	number of days
(.25)	0	16/24	8/24	0	0	0	0	0	probability
Tuesday		0	1	2	3	4	5	6	number of days
(.15)		0	16/24	8/24	0	0	0	0	probability
Wednesday			0	1	2	3	4	5	number of days
(.19)			0	16/24	8/24	0	0	0	probability
Thursday				0	1	2	3	4	number of days
(.20)				0	16/24	0	0	8/24	probability
Friday					0	1	2	3	number of days
(.21)					0	0	0	24/24	probability

Expected Total Float = 2+ 2/24 + (.25+.15+.19+.2) (16/24) (1) + (.25+.15+.19) (8/24) (2) + (.20) (8/24) (4) + (.21) (3) (24/24) = 3.900 days

Exhibit 2. Plains Availability Schedule

Available	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.	Mon.	Tues.	
Received Monday	0	1	2	3	4	5	6	7	8	number of days
(.25)	0	0	14/24	10/24	0	0	0	0	0	probability
Tuesday		0	1	2	3	4	5	6	7	number of days
(.15)		0	0	14/24	10/24	0	0	0	0	probability
Wednesday			0	1	2	3	4	5	6	number of days
(.19)			0	0	14/24	0	0	10/24	0	probability
Thursday				0	1	2	3	4	5	number of days
(.20)				0	0	0	0	24/24	0	probability
Friday					0	1	2	3	4	number of days
(.21)					0	0	0	14/24	10/24	probability

Expected Total Float = 2+ 2/24 + (.25+.15+.19) (14/24) (2) + (.25+.15) (10/24) (3) + (.19) (10/24) (5) + (.20) (4) + (.21) (14/24) (3) + (.21) (10/24) (4) = 5.185 days hence the incoming wire transfer charge assessed by the concentration bank must rightly be attributed to the non-concentration bank in the calculation of fixed charges.

Generally, payment for lock box charges may be on a cash basis or a compensating balance basis, or some combination of the two. The method of payment affects the cost to the corporation (in opportunity dollars) of maintaining a lock box at a particular bank, since this cost includes not only cash payments made to the bank, but also the cost of keeping compensating balances at the bank when in fact these balances could be invested elsewhere to earn additional funds.

We shall give two examples of the calculation of lock box charges. First, consider a lock box bank j^* that has a per check charge of \$.20 and a fixed charge of \$1,000 per year. If 1,000 checks are processed per year, the cost is 1,000 + .2(1,000) = \$1,200. If 2,000 checks are processed, the cost is \$1,400. Such a scheme may be easily incorporated into our model's objective function:

Minimize

$$\sum_{i \epsilon I} \sum_{j \epsilon J} c_{ij}x_{ij} + \sum_{j \epsilon J} d_j y_j + \sum_{i \epsilon I} \sum_{j \epsilon J} h_i s_j x_{ij} \qquad (9)$$

where d^*_j = 1,000 and s^*_j = .2, and where h_i is the expected number of checks received from customer zone i at lock box j^*.

A second example can be constructed using the charging scheme above, but with the requirement that compensating balances must be used to offset the charges. Suppose the bank allows a 7% earnings credit rate which is to be applied against 85% of collected balances. We also suppose that the corporation has an α = .08, and 1,000 checks are processed per year. For the fixed charge of \$1,000 and a per check charge of \$.20 we have the following calculations:

Equivalent Fixed Charge =

$$\frac{1{,}000}{(.85)(.07)}(.08) = \$1{,}344 \qquad (10)$$

Equivalent Variable Charge for 1 check =

$$\frac{(.20)}{(.85)(.07)}(.08) = \$.269. \qquad (11)$$

Thus, in our objective function d^*_j = \$1,344, and s^*_j = \$.269. (12)

Undertaking a Lock Box Study

A number of points must be considered before undertaking a lock box study. Probably the most important is the determination of sample checks to be used in the study. Since the opportunity costs of \$-float (the c_{ij} terms) are directly calculated from the sample checks, it is important to design the sampling process carefully. The problem of stratification by geographic area should be considered. In fact, the analyst should use a stratified approach in selecting a sample, if such geographic information is available. The cost of data collection generally demands that sample remittances from one month or less be used, but care should be taken to consider seasonality of corporate receipts. Finally, the size of the sample can be estimated for a given confidence interval and confidence level. Such sizing estimates are quite straightforward; they are outlined in detail in various statistics textbooks.

After a sample size has been decided upon and sampling undertaken, estimates for fixed and variable lock box costs must be made. Such estimates vary from corporation to corporation due to the specific lock box services required.

The Solution Algorithm

We have developed an efficient branch and bound algorithm to solve the lock box problem. (See [1] and [8] for a basic discussion of the branch and bound method.) The implicit exhaustive enumeration procedure inherent in the branch and bound process guarantees that an optimal solution is always found. This, of course, differs from earlier solution methods that generated "good," but not necessarily best, solutions. Also, we should say that the algorithm's computational efficiency depends on the number of potential lock boxes and not (generally) on the number of customer zones. This is due to the fact that the branch and bound process is applied to the lock box selection and not directly to customer zone selection.

The theoretical and computational underpinnings of the algorithm require a knowledge of discrete optimization theory as given in Geoffrion and Marsten [8]. An earlier working paper of ours [19] gives a detailed treatment of the development of the solution algorithm. The interested reader may refer to these sources for further details.

Computational Results — A Typical Lock Box Location Problem

Let us now consider the solution to a typical lock box location problem, where there are 46 potential lock box sites and 99 customer zones (in this case zip codes from which customer checks are received.) The first solution in Exhibit 3 is the optimal solution for the unconstrained lock box location problem (with no upper limit on the number of lock boxes allowed to be open in a solution). The optimal solution calls for seven lock box locations, with a resultant opportunity cost of dollar-float plus bank cash charges equal to $100,805. The solution indicates customer zones to be served by each of these seven banks, and it summarizes the various costs associated with each of the banks. Marginal cost associated with closing each of the seven lock boxes is also indicated. Following the optimal solution are three "next best" solutions based on a one-for-one swap of lock boxes that were open in the optimal solution. For example, in the first one-for-one swap solution, lock box #36 has been replaced by lock box #40, with an increase in the minimum objective function value to $100,810. Other one-for-one solutions have a similar interpretation.

Following the optimal solution, the lock box model determines a sequence of solutions in which the number of lock boxes open is successively reduced by one. For example, consider the solution shown in Exhibit 4 for a constrained problem where six lock boxes (one less than the unconstrained optimal solution) are allowed to be open. The minimum objective function value increases to $100,999. All the additional cost information is again provided, and the three next best solutions, based on a one-for-one swap of lock boxes that were open in the optimal six lock box solution, are indicated.

The process of reducing the number of lock boxes is then continued, until we are down to a one lock box solution. As can be seen in the summary in Exhibit 5, as the number of lock boxes allowed open is reduced, the (minimum) value of the objective function increases. Such an analysis allows the manager to determine the marginal cost associated with a fixed number of lock boxes.

Exhibit 6 presents a summary of our computational experience for a number of problems that have been solved to date. It depicts quite clearly the speed and efficiency of this model in determining solutions to problems that are quite large. These results are quite encouraging, because until recently it was thought that finding optimal solutions to such problems required an inordinate amount of central processing unit (CPU) time and was thus very costly. Our results suggest that even larger problems can be solved using a modest amount of CPU time.

Summary and Extensions

Several large banks and corporations are now using the lock box location model utilizing the branch and bound algorithm and the corresponding computer programs. We feel that this model offers an improved means of solving the lock box location problem in the context of more efficient cash management. Among the improvements are the following: 1) Improved calculation of expected total float; 2) Increased user flexibility: input options that allow the determination of solutions for several differing bank charging arrangements; 3) Improved output capability: output that provides the user with valuable sensitivity and incremental analysis information; 4) Greater computational efficiency: the ability to solve large problems (more than 45 potential lock boxes) using only a small amount of computer time (less than 40 seconds CPU time); and 5) Greater computational accuracy: the ability to always determine an optimal solution to the lock box location problem, with no restriction on the number of lock boxes allowed in solution.

We have also developed and implemented a similar model for remote disbursing account location analysis (see [3, 5, 8, 19, and 21] for details). The remote disbursing account location problem is essentially the converse of the lock box location problem. The objective of a remote disbursement system is to maximize the $-float associated with payments made to suppliers minus associated bank charges. We have solved several large-scale disbursement account location problems that are not presented here.

We are now working with several banks and corporations to implement the model using the latest bank availability schedules, bank charging schemes, and expected mail times. These programs, associated documentation, and illustrative examples are available from the authors.

References

1. Norman Agin, "Optimum Seeking With Branch and Bound," *Management Science* (December 1966), pp. B176–85.
2. R.L. Bulfin and V.E. Unger, "Computational Experience With An Algorithm For the Lock-Box Problem," *Proceedings of the 28th Association For Computing Machinery National Conference*, Atlanta, August 1973.
3. Robert F. Calman, *Linear Programming and Cash Management/Cash ALPHA*, Cambridge, MIT Press, 1968.

4. "Cash Management: The New Art of Wringing More Profit From Corporate Funds," *Business Week*, March 13, 1978, pp. 62–68.
5. F.F. Ciochetto, H.S. Swanson, J.R. Lee, and R.E.D. Woolsey, "The Lock Box Problem and Some Startling But True Computational Results For Large Scale Systems," Working Paper presented at the 41st National Meeting of the Operations Research Society of America, New Orleans, April 26–28, 1972.
6. Gerard Cornuejols, Marshall L. Fisher, and George L. Nemhauser, "Location of Bank Accounts to Optimize Float: An Analytic Study of Exact and Approximate Algorithms," *Management Science* (April 1977), pp. 789–810.
7. L.B. Ellwein, "Fixed Charge Location–Allocation Problems with Capacity and Configuration Constraints," Technical Report No. 70-2, Department of Industrial Engineering, Stanford University, August 1970.
8. A.M. Geoffrion and R.E. Marsten, "Integer Programming Algorithms: A Framework and State-of-the-Art Survey," *Management Science* (May 1972), pp. 465–91.
9. Lawrence J. Gitman, Keith D. Forrester, and John R. Forrester, "Maximizing Cash Disbursement Float," *Financial Management* (Summer 1976), pp. 15–24.
10. "How Business Lives Beyond Its Means," *Business Week*, November 15, 1969, pp. 72*ff.*
11. Robert L. Kramer, "Analysis of Lock-box Locations," *Bankers Monthly Magazine*, May 15, 1966, pp. 36–40.
12. Alan Kraus, Christian Janssen, and Alan K. McAdams, "The Lock-box Location Problem," *Journal of Bank Research* (Autumn 1970), pp. 51–58.
13. Ferdinand K. Levy, "An Application of Heuristic Problem Solving to Accounts Receivable Management," *Management Science* (February 1966), pp. B236–244.
14. Alan K. McAdams, "Critique of: A Lock-box Model," *Management Science* (October 1968) pp. 888–90.
15. S.F. Maier and J.H. Vander Weide, "The Lock-Box Location Problem: A Practical Reformulation," *Journal of Bank Research* (Summer 1974), pp. 92–95.
16. S.F. Maier and J.H. Vander Weide, "A Unified Location Model for Cash Disbursements and Lock-Box Collections," *Journal of Bank Research* (Summer 1976), pp. 166–72.
17. "Making Millions by Stretching the Float," *Business Week*, November 23, 1974, pp. 89–90.
18. R.M. Nauss, "An Improved Algorithm for the Capacitated Facility Location Problem," forthcoming in *Operational Research Quarterly*.
19. R.M. Nauss and R.E. Markland, "Real World Experience with an Optimal Lock Box Location Algorithm," Working Paper presented at TIMS/ORSA Joint National Meeting, San Francisco, May 1977.
20. Gerald A. Pogue, Russell B. Faucett, and Ralph N. Bussard, "Cash Management: A Systems Approach," *Industrial Management Review* (Winter 1970), pp. 55–73.
21. G. Sá, "Branch-and-Bound and Approximate Solutions to the Capacitated Plant-Location Problem," *Operations Research* (November-December 1969), pp. 1005–16.
22. R.J. Shanker and A.A. Zoltners, "The Corporate Payments Problem," *Journal of Bank Research* (Spring 1972), pp. 47–53.
23. R.J. Shanker and A.A. Zoltners, "An Extension of the Lock Box Location Problem," *Journal of Bank Research* (Winter 1972), p. 62.
24. James McN. Stancill, "A Decision Rule Model for the Establishment of a Lock-box," *Management Science* (October 1968), pp. 884–87.
25. William J. Tallent, "Cash Management: A Case Study," *Management Accounting* (July 1974), pp. 20–24.

Exhibit 5. Summary Table–Lock Box Solutions

Solution #	Description	Value of Objective Function	Lock Boxes In Solution
1	Unconstrained*–7 Lock Boxes	$100,805	1, 10, 33, 36, 37, 41, 43
2	6 Lock Boxes	$100,999	1, 10, 33, 37, 41, 43
3	5 Lock Boxes	$101,398	1, 10, 33, 41, 43
4	4 Lock Boxes	$102,735	1, 33, 41, 43
5	3 Lock Boxes	$104,116	1, 33, 41
6	2 Lock Boxes	$107,126	1, 41
7	1 Lock Box	$123,257	10

*Optimal solution to original unconstrained problem.

Exhibit 3. Optimal Solution–Unconstrained Lock Box Problem

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          LOCK BOX OPTIMIZATION MODEL

FIGURES ARE ON AN ANNUAL BASIS UNLESS OTHERWISE NOTED

CORPORATE INTERNAL RATE OF RETURN=  0.060
THIS SOLUTION IS CONSTRAINED TO HAVE NO MORE THAN  46 LOCK BOXES OPEN


BANKS IN SOLUTION
AND ZIP CODES SERVED
                                      AVERAGE..DISCOUNTED.. EXPECTED..VARIABLE..VARIABLE..FIXED.....FIXED.....TOTAL.....TOTAL
                                      $-FLOAT..$-FLOAT..... CHECKS....CHARGES...COMP BAL .CHARGES...COMP BAL .CHARGES...COMP BAL
                                      PER DAY..PER DAY..... PER YEAR..PER YEAR..PER MONTH.PER YEAR..PER MONTH.PER YEAR..PER MONTH

**************************

BANK       1
ZIP CODES    1    2    3    4    5
             6    8    9   11   12
            13   16   17   19   20
            24   25   26   33   34
            38   43   45   48   49
            51   53   54   55   56
            57   58   59   60   61
            63   64   65   67   69
            70   72   73   74   76
            78   79   80   81   82
            83   84   85   86   87
            88   89   90   91   92
            93   95   96   97   98
            99
                                      915216.   54913.      960       96.     2000.     1000.    20833.     1096.    22833.
MARGINAL COST OF CLOSING
LOCK BOX IS       4309.
**************************

BANK      10
ZIP CODES    7   10   14   15   46
            47   77
                                      161592.    9696.      125       13.      245.     1000.    19583.     1013.    19828.
MARGINAL COST OF CLOSING
LOCK BOX IS       1326.
**************************

BANK      33
ZIP CODES   21   28   29   37   50
            52   68
                                       95158.    5710.      110       16.      357.     1000.    21667.     1016.    22024.
MARGINAL COST OF CLOSING
LOCK BOX IS       1215.
**************************

BANK      36
ZIP CODES   22   27   66   75
                                       34073.    2044.       70        6.      107.     1000.    19167.     1006.    19274.
MARGINAL COST OF CLOSING
LOCK BOX IS        194.
**************************
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BANK 37 ZIP CODES 18 23 44 94 MARGINAL COST OF CLOSING LOCK BOX IS 220.	29017.	1741.	25	3.	63.	1000.	21000.	1003.	21063.
BANK 41 ZIP CODES 30 31 32 39 40 41 42 62 MARGINAL COST OF CLOSING LOCK BOX IS 8855.	287316.	17239.	130	13.	271.	1000.	20833.	1013.	21104.
BANK 43 ZIP CODES 35 36 71 MARGINAL COST OF CLOSING LOCK BOX IS 1381.	38450.	2307.	60	9.	195.	1000.	21667.	1009.	21862.
TOTALS FOR LOCK BOX SYSTEM	1560819.	93649.	1480	156.	3238.	7000.	144750.	7156.	147988.

TOTAL DISCOUNTED $-FLOAT 93649.
TOTAL BANK CASH CHARGES 7156.
DISCOUNTED $-FLOAT PLUS BANK CASH CHARGES 100805.

NOTE: MARGINAL COST OF CLOSING A LOCK BOX IS COMPUTED BY ADDING THE ADDITIONAL DISCOUNTED FLOAT INCURRED IF THE LOCK BOX WERE CLOSED (AND THE LOCK BOXES IN THIS SOLUTION REMAIN OPEN) AND SUBTRACTING THE FIXED AND VARIABLE CHARGES INCURRED BY THE LOCK BOX

3 NEXT BEST SOLUTIONS BASED ON ONE-FOR-ONE SWAP OF BANKS OPEN IN OPTIMAL SOLUTION.
(BANKS FIXED OPEN OR CLOSED ARE NOT INCLUDED)

TOTAL DISCOUNTED $-FLOAT PLUS BANK CHARGES= 100810.00

BANKS IN SOLUTION ARE

1
10
33
37
40
41
43

TOTAL DISCOUNTED $-FLOAT PLUS BANK CHARGES= 101105.00

BANKS IN SOLUTION ARE

1
10
33
36
40
41
43

TOTAL DISCOUNTED $-FLOAT PLUS BANK CHARGES= 100951.00

BANKS IN SOLUTION ARE

1
10
33
37
41
43
45

Exhibit 4. Optimal Solution–Constrained Lock Box Problem

```
             LOCK BOX OPTIMIZATION MODEL

FIGURES ARE ON AN ANNUAL BASIS UNLESS OTHERWISE NOTED

CORPORATE INTERNAL RATE OF RETURN=  0.060
THIS SOLUTION IS CONSTRAINED TO HAVE NO MORE THAN   6 LOCK BOXES OPEN


BANKS IN SOLUTION
AND ZIP CODES SERVED
                          AVERAGE..DISCOUNTED...EXPECTED..VARIABLE..VARIABLE..FIXED.....FIXED.....TOTAL.....TOTAL
                          $-FLOAT..$-FLOAT......CHECKS....CHARGES...COMP BAL .CHARGES...COMP BAL .CHARGES...COMP BAL
                          PER DAY..PER DAY......PER YEAR..PER YEAR..PER MONTH.PER YEAR..PER MONTH.PER YEAR..PER MONTH

*************************
BANK          1
ZIP CODES    1   2   3   4   5
             6   8   9  11  12
            13  16  17  19  20
            22  24  25  26  33
            34  38  43  45  48
            49  51  53  54  55
            56  57  58  59  60
            61  63  64  65  66
            67  69  70  72  73
            74  75  76  78  79
            80  81  82  83  84
            85  86  87  88  89
            90  91  92  93  95
            96  97  98  99
                           942249.   56535.     1010       101.     2104.     1000.    20833.    1101.    22938.
MARGINAL COST OF CLOSING
LOCK BOX IS        4519.
*************************

BANK         10
ZIP CODES    7  10  14  15  46
            47  77
                           161592.    9696.      125        13.      245.     1000.    19583.    1013.    19828.
MARGINAL COST OF CLOSING
LOCK BOX IS        1326.
*************************

BANK         33
ZIP CODES   21  27  28  29  37
            50  52  68
                           122058.    7324.      130        19.      422.     1000.    21667.    1019.    22089.
MARGINAL COST OF CLOSING
LOCK BOX IS        1309.
*************************

BANK         37
ZIP CODES   18  23  44  94
                            29017.    1741.       25         3.       63.     1000.    21000.    1003.    21063.
MARGINAL COST OF CLOSING
LOCK BOX IS         399.
*************************
```

```
BANK     41
ZIP CODES   30   31   32   39   40
            41   42   62
                              287316.    17239.    130    13.    271.    1000.    20833.    1013.    21104.
MARGINAL COST OF CLOSING
LOCK BOX IS        8855.
**************************

BANK     43
ZIP CODES   35   36   71
                              38450.     2307.     60     9.     195.    1000.    21667.    1009.    21862.
MARGINAL COST OF CLOSING
LOCK BOX IS          1381.


TOTALS FOR LOCK BOX SYSTEM    1580679.   94841.    1480   158.   3300.   6000.    125583.   6158.    128884.

TOTAL DISCOUNTED $-FLOAT                     94841.
TOTAL BANK CASH CHARGES                       6158.
DISCOUNTED $-FLOAT PLUS BANK CASH CHARGES   100999.

NOTE: MARGINAL COST OF CLOSING A LOCK BOX IS COMPUTED BY ADDING THE ADDITIONAL DISCOUNTED FLOAT INCURRED IF THE
LOCK BOX WERE CLOSED (AND THE LOCK BOXES IN THIS SOLUTION REMAIN OPEN) AND SUBTRACTING THE FIXED AND VARIABLE CHARGES
INCURRED BY THE LOCK BOX

3 NEXT BEST SOLUTIONS BASED ON ONE-FOR-ONE SWAP OF BANKS OPEN IN OPTIMAL SOLUTION.
(BANKS FIXED OPEN OR CLOSED ARE NOT INCLUDED)


TOTAL DISCOUNTED $-FLOAT PLUS BANK CHARGES= 101025.00

BANKS IN SOLUTION ARE

  1
 10
 33
 36
 41
 43


TOTAL DISCOUNTED $-FLOAT PLUS BANK CHARGES= 101172.00

BANKS IN SOLUTION ARE

  1
 10
 33
 41
 43
 45


TOTAL DISCOUNTED $-FLOAT PLUS BANK CHARGES= 101389.00

BANKS IN SOLUTION ARE

  1
 10
 33
 41
 42
 43
```

Exhibit 6. Summary of Computational Results

Problem Type	Number of Potential Locations	Number of Customer Zones	Number of Locations in Optimal Solution	Value of Objective Function	CPU Time in Seconds*
Lock Box	15	115	6	143,382	4.4
Lock Box	15	99	5	145,016	7.2
Lock Box	47	120	24**	199,134	36.5
Lock Box	47	120	26**	202,861	30.1
Lock Box	45	31	2	46,876	1.4
Lock Box	46	99	7	100,805	35.1
Lock Box	46	100	10	58,063	4.5
Lock Box	46	100	10	129,416	36.4
Lock Box	46	100	9	81,531	12.2
Lock Box	92	100	7	98,748	38.7
Lock Box	94	120	6	227,013	32.3
Lock Box	45	101	1	14,326	10.9

*Includes input/output time for runs made on IBM 370/168.

**Problems involving very small fixed charges, thus resulting in a large number of open lock boxes in the optimal solution.